

# Update on Semileptonic Charm Decays

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A brief update is given on recent developments in the theory of exclusive semileptonic charm decays. A check on analyticity arguments from the kaon system is reviewed. Recent results on form factor shape measurements are discussed.

## 1. Introduction

Semileptonic meson decays provide a valuable arena to study the weak and strong interactions. On the one hand, once the effects of the strong interaction are under control, weak mixing parameters ( $V_{ub}$ ,  $V_{us}$ , ...) can be extracted from the overall normalization. On the other hand, if these mixing parameters are taken from other processes, then the semileptonic rates probe complicated underlying strong dynamics, thus yielding an important test for lattice methods [1] and for our understanding of nonperturbative QCD.

Apart from the overall normalization, the spectral shapes are also interesting [2]. The energy spectra (the “ $q^2$  dial”) are governed by quark-hadron duality, and are constrained by dispersion relations and analyticity. Shape parameters for different quark masses (the “ $m$  dial”) can be interpreted by means of effective field theory descriptions in the appropriate regimes of validity. These parameters provide inputs to other processes governed by the same effective field theory. For example, the form factors measured in  $B \rightarrow \pi \ell \nu$  constrain  $B \rightarrow \pi \pi$ .

The charm quark holds a privileged position in this scheme. Abundant experimental data exist for  $D \rightarrow K$  and for  $D \rightarrow \pi$  transitions. The charm mass is heavy enough to be treated using heavy-quark methods on the lattice, but light enough so that the full range of physically allowed momentum transfers are accessible in present simulations [3]. Charm decays thus provide a powerful test of lattice QCD methods that can be applied to other heavy-meson systems.

Charm decays are important for testing another important, but perhaps less well-known aspect of QCD, namely the constraints imposed by analyticity [4, 5, 6, 7, 8]. These constraints imply a convergent expansion in powers of a small parameter that measures the distance between the physically allowed kinematic region, and the region of resonances and production thresholds. While the existence of this small parameter is well known, the usefulness of the expansion has been practically negated by appeals to “unitarity bounds”. It has only recently become clear with the advent of rigorous power-counting arguments that we can “take seriously” the expansion provided by analyticity [7]. Charm decays provide a valuable illustration and crosscheck of these arguments.

Table I Maximum  $|z|$  throughout semileptonic range (from [2]).

Process	CKM element	$ z _{\max}$
$\pi^+ \rightarrow \pi^0$	$V_{ud}$	$3.5 \times 10^{-5}$
$B \rightarrow D$	$V_{cb}$	0.032
$K \rightarrow \pi$	$V_{us}$	0.047
$D \rightarrow K$	$V_{cs}$	0.051
$D \rightarrow \pi$	$V_{cd}$	0.17
$B \rightarrow \pi$	$V_{ub}$	0.28

An overview of these ideas for general semileptonic meson transitions has been presented previously in [2]; further discussion and references may be found there. The focus in the present report is on some recent illustrations from kaon physics that are relevant to charm physics, and on an update of shape measurements in the charm system.

The remainder of the talk is organized as follows. Section 2 outlines the constraints of analyticity, and describes the explicit test of power-counting afforded by  $K_{\ell 3}$  decays. Section 3 tabulates recent results on the shape parameters in  $D \rightarrow K$  and  $D \rightarrow \pi$  semileptonic transitions. Section 4 concludes with a discussion of the relevance of these shape parameters for testing lattice QCD, and for applying factorization in  $B$  decays.

## 2. Analyticity and simplicity

The mere existence of a field theory description of the physical hadrons, and their weak current probes, implies powerful constraints on form factors. In particular, singularities in hadronic transition amplitudes are determined by kinematics. Analyticity translates into a convergence expansion in a small variable once the domain of analyticity is mapped onto a standard region.

In practical terms, this mapping is simply a rearrangement of the series expansion of the form factor,

$$F(t) = F(0) + \dots \quad (1)$$

For example, a simple pole model of the form factor

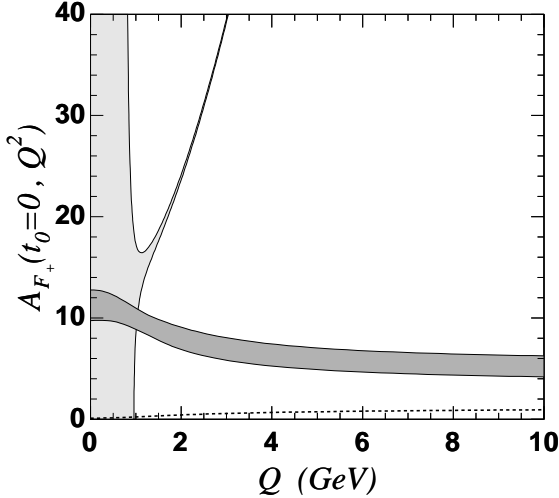


Figure 1: Sum of squares of expansion coefficients (dark) compared to unitarity bound (light). (from [9])

would “resum” into the form

$$F(t)/F(0) = 1 + t/m^2 + (t/m^2)^2 + \dots = \frac{1}{1 - q^2/m^2}. \quad (2)$$

Without knowing what the form factor “resums” into, analyticity implies that the series has to rearrange itself into the form

$$F(t)/F(0) = 1 + a_1 z(t) + a_2 z^2(t) + \dots, \quad (3)$$

where  $a_i$  are  $\mathcal{O}(1)$  in a rigorous sense ( $\sum_i a_i^2$  is also  $\mathcal{O}(1)$ ), and  $z$  is a variable bounded by the distance of the physical region from singularities.

The smallness of  $|z|$  in (3) implies that terms beyond linear order are highly suppressed. There is thus an essentially unique choice of shape parameter that unites semileptonic transitions from various decay modes—the slope of the form factor, say at  $q^2 = 0$ . It turns out that the numerical value of this parameter is in itself an interesting quantity. It provides a quantitative test of lattice versus experimental shape; it is an input to related hadronic processes; and it probes an unsolved question in the QCD dynamics governing form factors [2].

The tools of analyticity are well-known, but their usefulness has not been appreciated, due to a reliance on unitarity as the only means to bound the coefficients  $a_k$ . In fact, rigorous power counting arguments show that both  $a_k$  and  $\sum_k a_k^2$  are order unity, even when large ratios of scales, such as  $m_Q/\Lambda_{\text{QCD}}$ , are present ( $m_Q$  a heavy-quark mass, and  $\Lambda$  a hadronic scale). It is important to test and make full use of this expansion when dealing with complicated QCD systems.

Analysis of  $K_{\ell 3}$  decays ( $K \rightarrow \pi \ell \nu$ ) provides a revealing confirmation of these ideas. The analysis also provides an important constraint on the determination of  $|V_{us}|$  [9]. Figure 1 compares the unitarity bound to the actual value of the sum of squares of coefficients, as calculated from ALEPH  $\tau$  decay data [10]. The result is plotted for a range of values of the OPE parameter  $Q$  ( $Q$  corresponds to the virtuality of the produce meson pair in the crossed channel; large  $Q$  implies that the OPE is increasingly reliable). It can be seen that the unitarity bound begins to wildly overestimate the possible size of the coefficients for even moderately large  $Q$ . The convergence of the series improves as  $Q$  becomes larger, since the sensitivity to the  $K^*$  pole is lessened; reliance on the unitarity bound would however force us to work at small  $Q$ .

### 3. Shape parameters

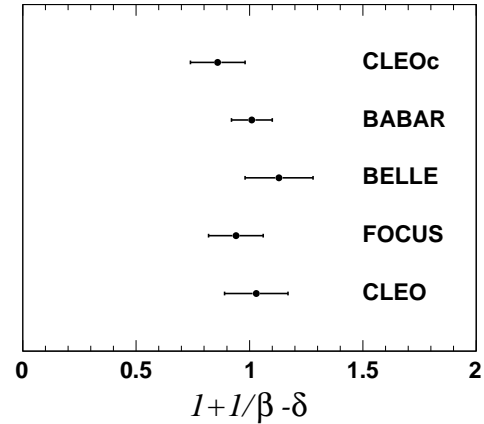


Figure 2: Relative slope of the vector form factor in  $D \rightarrow K \ell \nu$ . As tabulated in (8) and [2], from CLEO [11], FOCUS [12], BELLE [13], BABAR [14] and CLEOc [15].

Given the rapid convergence of the series (3), it turns out that no more than a normalization and slope can be measured in present experiments [2]. It is convenient to define the slope at  $q^2 = 0$ , in which case we have:

$$\begin{aligned} \frac{1}{\beta} &\equiv \frac{m_H^2 - m_L^2}{F_+(0)} \frac{dF_0}{dt} \Big|_{t=0}, \\ \delta &\equiv 1 - \frac{m_H^2 - m_L^2}{F_+(0)} \left( \frac{dF_+}{dt} \Big|_{t=0} - \frac{dF_0}{dt} \Big|_{t=0} \right) \\ &\equiv \frac{F_+(0) + F_-(0)}{F_+(0)}. \end{aligned} \quad (4)$$

Here  $F_+$  is the vector form factor that dominates for massless leptons in pseudoscalar-pseudoscalar transi-

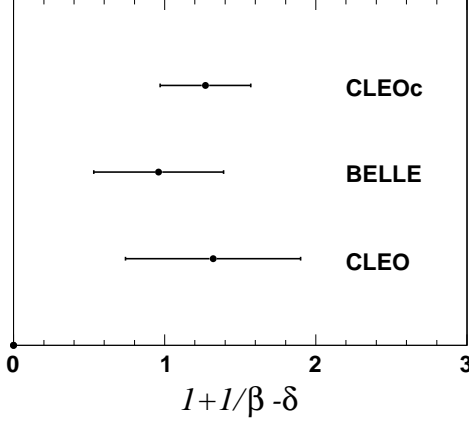


Figure 3: Relative slope of the vector form factor in  $D \rightarrow \pi \ell \nu$ . As tabulated in (9) and [2], from CLEO [11], BELLE [13], and CLEOc [15]

tions, and  $F_0$  is the scalar form factor, accessible for massive leptons, or in lattice simulations.

In the heavy-quark limit of heavy-to-light form factors, soft-collinear effective theory [16] can be used to extract the heavy-quark mass dependence, and organize form-factor contributions in powers of  $\Lambda_{\text{QCD}}/E$ , where  $E$  is the energy of the outgoing light hadron. At leading power [17],

$$F_+(E) = \sqrt{m_H} \left[ \zeta(E) + \left( \frac{4E}{m_H} - 1 \right) H(E) \right],$$

$$\frac{m_H}{2E} F_0(E) = \sqrt{m_H} [\zeta(E) + H(E)]. \quad (5)$$

Here  $\zeta$  is the so-called soft overlap contribution to the form factor, and  $H$  is the hard-scattering component, due to hard gluon exchange with the spectator quark. Apart from scaling violations, the functions  $\zeta$  and  $H$  both have an energy dependence  $\zeta \sim H \sim E^{-2}$ . Then independent of any model assumptions,

$$\frac{1}{\beta} = - \left. \frac{d \ln(\zeta + H)}{d \ln E} \right|_{E=m_H/2} - 1 + \mathcal{O}(\alpha_s, \Lambda/m_H), \quad (6)$$

and

$$\delta(m_L, m_H) = \left. \frac{2H}{\zeta + H} \right|_{E=m_H/2} + \mathcal{O}(\alpha_s, \Lambda/m_H) \quad (7)$$

Thus  $1/\beta - 1$  measures scaling violations, and  $\delta$  measures the size of hard-scattering contributions.

From the recent BABAR [14] and CLEOc [15] results on  $D \rightarrow K$ ,

$$D \rightarrow K :$$

$$1 + 1/\beta - \delta = 1.01 \pm 0.04 \pm 0.08 \quad [14]$$

$$0.85 \pm 0.06 \pm 0.10 \quad [15]. \quad (8)$$

To compare with [2], the central values and first error correspond to keeping just the linear term ( $a_0$  and  $a_1$ ) in (3), and the second error is a conservative bound on residual shape uncertainty from allowing  $a_2$  with  $|a_2/a_0| \leq 10$ . Together with previous CLEO, FOCUS and BELLE measurements, these values are displayed in Figure 2. A naive average of these results yields  $1 + 1/\beta - \delta = 0.97 \pm 0.05$  ( $\chi^2 = 2.5$  for 5-1 d.o.f.).

From the CLEOc results on  $D \rightarrow \pi$ ,

$$D \rightarrow \pi :$$

$$1 + 1/\beta - \delta = 1.27 \pm 0.13 \pm 0.27 \quad [15]. \quad (9)$$

Together with previous CLEO and BELLE measurements, these values are displayed in Figure 3. A naive average of these results yields  $1 + 1/\beta - \delta = 1.19 \pm 0.23$  ( $\chi^2 = 0.4$  for 3-1 d.o.f.).

## 4. Conclusions

Semileptonic charm decays provide an important arena in which to test lattice QCD. Unfortunately, it is not possible to definitively test the experimental shape predictions against unquenched lattice simulations at present, since the lattice results are so far reported only in terms of model parameters, that need not agree between theory and experiment, or between experiments with different acceptance and systematics. It is interesting to note that while some model parameters show poor agreement between experiments [18], there is no obvious discrepancies among the physical results shown in Figure 2.

The actual value of the shape parameters are also of interest. For example, the parameter  $\delta$  is an important input to factorization analyses of  $B \rightarrow \pi\pi$  decays [19], usually phrased in terms of an (inverse) moment of the  $B$  meson wavefunction,  $\lambda_B$ :

$$\delta(m_\pi, m_B) = \frac{6\pi C_F}{N_c} \frac{f_B f_\pi \alpha_s}{m_B \lambda_B F_+(0)} + \dots \quad (10)$$

A plausible conjecture of monotonicity for  $\delta$  implies that  $\delta(m_\pi, m_B) < 0.35 \pm 0.03$ . A much larger value of  $\delta(m_\pi, m_B)$  in  $B \rightarrow \pi$ , e.g.  $\delta \approx 1$  [20], would require a dramatic behavior of this physical quantity as a function of quark mass, a behavior that should already be apparent in the charm system. For example, we would expect  $\delta(m_\pi, m_D) > \delta(m_K, m_D)$  (see Figure 8 of [2]). It is exciting that current lattice simulations can probe this quantity in both the charm and bottom systems, not by evaluating moments of  $B$  meson wavefunctions, but by computing form factor slopes. Experimental measurements are sensitive to the combination  $1/\beta - \delta$ , so that the scalar form factor slope (denoted  $1/\beta$ ) is the most urgent requirement from the lattice. Again, definitive conclusions must await a model-independent presentation of simulation results.

## Acknowledgments

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